SR-001 TECHNICAL APPLICATION NOTE

Mode-Matching to the SuperCavity™

Introduction

The application at Newport of ring laser gyro mirror technology to the Fabry Perot Interferometer has resulted in a state-of-the-art interferometric instrument—the SuperCavity. This compact interferometer has an exceptional efficiency and an extremely high finesse which allows it to be used for detailed investigations and analysis of coherent source mode structures in a near-real time environment.

Of great importance in the use of this kind of instrument is the injection of the source into the interferometer. Because SuperCavity is a non-confocal, spherically symmetric cavity, the incoming beam from the source must be both well-aligned and mode-matched to the fundamental cavity mode of the FabryPerot. Calculation of the lens focal length required to most efficiently mode-match is the subject presented in this Technical Application Note. Please see the SuperCavity Manual and its references for the Theory of Operation and the theoretical discussion on Fabry-Perot interferometers.

Basic Equations

The Cavity mode-matching conditions are based on the propagation and focusing properties of Gaussian beams. In the paraxial approximation, Gaussian beam waists are imaged by lenses in a manner similar to that of geometrical optics. In general, the law describing the change of waveform curvature by a lens is given by:

$$\frac{1}{q_2} = \frac{1}{q_1} - \frac{1}{f} \tag{1}$$

where the complex radius of waveform curvature is defined as:

$$q_n(z) = z_n + i\left\{\frac{\pi w_n^2}{\lambda}\right\} \quad \text{or} \tag{2}$$

$$q_n(z) = z_n + ia_n, \qquad n = 1,2 \tag{3}$$

and where w_n is the waist of the beam, $\pi = 3.14159$, and $a_n = (\pi w_n^2)/\lambda$. z is the distance from the point of interest, i.e. the lens, to the waist, as shown in Figure 1.

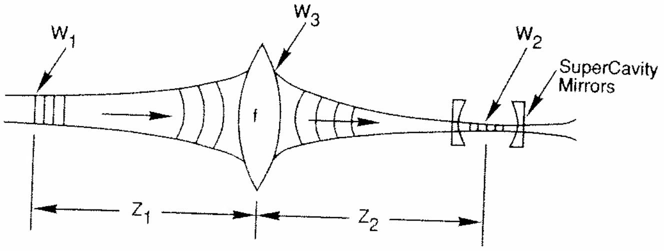

Figure 1. Mode Matching

Inserting Equation (3) into Equation (1), and separating the real and imaginary parts, the following two cavity matching conditions are derived:

$$a_2 = \frac{f^2 a_1}{(z_1 - f)^2 + a_1^2} \tag{4}$$

$$z_2 = -f\left\{\frac{a_1^2 + z_1(z_1 - f)}{a_1^2 + (z_1 - f)^2}\right\} \tag{5}$$

The first condition establishes the required focal length, f, of the coupling lens, while the second condition gives its location, z_2 with respect to the SuperCavity and to the input beam.

Using Equations (4) and appropriate values for the 1" and 20 micron SuperCavities we get $a_2 = 61$ mm and 1.7 mm, respectively. a_1 depends on the source laser waist w_1. In most cases a_1 varies between 50 mm for $w_1 = .1$ mm and 1000 mm for $w_1 = .5$ mm.

Special Cases

A. $z_1 >> f$ and a_1, Far-Field Approximation:

This is the most common case, where the coupling lens is located far from the laser. From Equations (4) and (5), and assuming $z_1 >> f$ and $z_1 >> a_1$, then:

$$f = \frac{w_2 z_1}{w_1} \tag{6}$$

$$z_2 = -f \tag{7}$$

The Equation for guassian propagation of beam waists, (See the Optics Tutorial Section of the Newport Catalog, or any Optics reference book) is:

$$w_3^2 = w_1^2\left\{1 + \left(\frac{\lambda z_1}{\pi w_1^2}\right)^2\right\}.$$

For large z_1, then, $w_3 = (\lambda z_1)/(\pi w_1)$,where w_3 is half the spot diameter at the lens, as in Figure 1. Using this in (6) gives:

$$f = \frac{\pi w_2}{\lambda} w_3 \tag{8}$$

Cavity matching conditions are given by Equations (7) and (8). These conditions dictate that the SuperCavity is located at the focal point of the lens, and that the focal length of the coupling lens is given by equation (8).

Condition (8) is plotted in Figure 2 for both the 1" and the 20 micron cavities. The horizontal axis is the operating wavelength, while the vertical axis is the focal length (in mm) of the coupling lens for an input beam with a mode diameter of 1 mm (D = 1). For mode diameters other than 1 mm, the focal length of the coupling lens f_D is simply $f_D = Df_{(D=1)}$, where D is the number of mm of the mode diameter.

For example, consider a laser source of wavelength 620 nm and a spot size of 1.5 mm at the lens. The required focal length for the coupling lens is 415 mm and 70 mm for SR-100 and SR-200 series SuperCavities, respectively. The corresponding distance from the laser to the lens should be at least 2 m and 0.4 m, respectively.

As the wavelength increases, the divergence angle of the source also increase (w_1 = constant). Hence, for a constant z_1 and w_1, the spot size at the mirror becomes larger; therefore, a coupling lens of a shorter focal length is needed for efficient coupling. This trend is also shown in Figure 2.

B. $z_1 = f$, Intermediate-Field Approximation:

This is a special intermediate point. From Equations (4) and (5), and using $z_1 = f$ then:

$$f = \frac{\pi w_2 w_1}{\lambda} \tag{9}$$

$$z_2 = -f \tag{10}$$

Equations (9) and (10) are identical to (7) and (8) except that w_3 in Equation (9) is replaced by the waist w_1. Here the waist of the source laser must be known. Because this is not in the far field region, the beam has not had a chance to expand, and the spot size at the cavity waist (w_2) is not much bigger than the beam waist (w_1).

In this case, Equation (8) overestimates the required focal length by the ratio w_3/w_1, i.e. by the amount of growth of the spot size as compared to the beam waist. A more accurate estimate may be obtained by using Figure 2 and the same $f_D = D\, f_{(D=1)}$ formula except that the value used for the spot size is now twice the beam waist.

Note that the variations in the lens location (i.e. z_1) may lead to appreciable variations in z_2 and in the lens focal length. For example, setting $z_1 = f + a_1$, we get $z_2 = -f\,(1 + z_1/a_1)/2$ and $f = \sqrt{2a_1a_2}$. This leads to a change in the location of the SuperCavity waist and in the lens focal length.

C. $z_1 << f$, Near-Field Approximation:

In this third case, the Fabry-Perot interferometer is located very close to the laser under examination. From Equations (4) and (5) we get:

$$f = \frac{\pi w_2}{\lambda}\left\{\frac{w_1}{\sqrt{1-\left(\frac{w_2}{w_1}\right)^2}}\right\} \tag{11}$$

$$z_2 = -f\left\{\frac{a_1^{\,2} - f z_1}{a_1^{\,2} + f^2}\right\} \tag{12}$$

1" Cavities (SR-100):

For the SR-100 series SuperCavities, the waist w_2 is in the range of 0.1 to 0.17 mm. This waist might be comparable to the input laser's waist w_1. Therefore, the required focal length for the lens as given by Equation (11) varies by a large margin depending on the ratio of w_2 to w_1.

20 micron Cavities (SR-200):

Typically, the waists for these cavities are 16 to 30 microns, and therefore, $w_2 << w_1$. Since the incoming laser beam has not diverged (especially if the output mirror of the laser is a flat coupler) then the spot size at the coupling lens is close to the laser's waist w_1. Therefore Equation (11) may be reduced to the much simpler form of Equation (8).

Equation (12) may also be reduced to (7); however, this requires additional limits, namely, $a_1^2 >> f^2$, and $a_1^2 >> fz_1$. These conditions are not difficult to implement and check, since the lens focal length is typically in the range of 50 to 150 mm and a_1 can easily reach 1000 mm.

Summary

Mode matching conditions for 20 micron SuperCavities are given by Equations (7) and (8) and are shown in Figure 2 for both the near field and the far field limit.

Matching conditions for 1" cavities are given by Figure 2 for the far field limit. In the near field limit the conditions are given by Equations (11) and (12). No figure is generated for this case due to the sensitivity of the focal length to the ratio w_2/w_1.

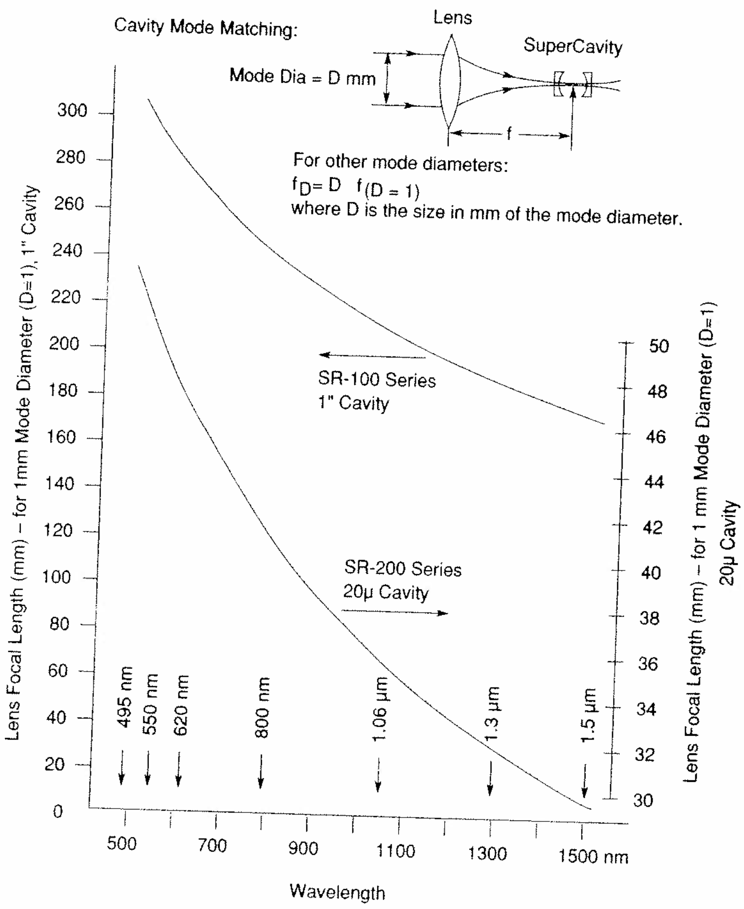

Figure 2. Cavity Mode Matching

The Required lens focal length is given for both 1" and 20μm mirror spacings and for an input mode diameter of 1mm, assuming far field conditions ($z_1 >> f$). For other mode diameters, $f_D = Df_{(D=1)}$.

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